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HW 2 – Part 1

Solutions

1 (a)

$$D+E=\begin{pmatrix}1 & 5 & 2\\ -1 & 0 & 1\\ 3 & 2 & 4\end{pmatrix}+\begin{pmatrix}6 & 1 & 3\\ -1 & 1 & 2\\ 4 & 1 & 3\end{pmatrix}=\begin{pmatrix}7 & 6 & 5\\ -2 & 1 & 3\\ 7 & 3 & 7\end{pmatrix}$$

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1 (b)

$$5A=5\begin{pmatrix}3 & 0\\ -1 & 2\\ 1 & 1\end{pmatrix}=\begin{pmatrix}15 & 0\\ -5 & 10\\ 5 & 5\end{pmatrix}$$

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1 (c)

$$2B-C=2\begin{pmatrix}4 & -1\\ 0 & 2\end{pmatrix}-\begin{pmatrix}1 & 4 & 2\\ 3 & 1 & 5\end{pmatrix}$$

$$=\underbrace{\begin{pmatrix}8 & -2\\ 0 & 4\end{pmatrix}}_{2\times 2}+\underbrace{\begin{pmatrix}-1 & -4 & -2\\ -3 & -1 & -5\end{pmatrix}}_{2\times 3}$$

This sum is not defined because the matrices are not of the same size

1 (d)

$$4E-2D = 4\begin{pmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{pmatrix} - 2\begin{pmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 24 & 4 & 12 \\ -4 & 4 & 8 \\ 16 & 4 & 12 \end{pmatrix} + \begin{pmatrix} -2 & -10 & -4 \\ 2 & 0 & -2 \\ -6 & -4 & -8 \end{pmatrix}$$

$$= \begin{pmatrix} 22 & -6 & 8 \\ -2 & 4 & 6 \\ 10 & 0 & 4 \end{pmatrix}$$

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1 (e)

$$-3(D+2E) = -3\left[\begin{pmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{pmatrix} + 2\begin{pmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{pmatrix}\right]$$

$$= -3\left[\begin{pmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 12 & 2 & 6 \\ -2 & 2 & 4 \\ 8 & 2 & 6 \end{pmatrix}\right]$$

$$= -3\begin{pmatrix} 13 & 7 & 8 \\ -3 & 2 & 5 \\ 11 & 4 & 10 \end{pmatrix} = \begin{pmatrix} -39 & -21 & -24 \\ 9 & -6 & -15 \\ -33 & -12 & -30 \end{pmatrix}$$

1 (f)

$$A - A = \begin{pmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} = O_{3\times 2}$$

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1 (g)

$$A + B = \underbrace{\begin{pmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{pmatrix}}_{3\times 2} + \underbrace{\begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix}}_{2\times 2}$$

This sum is undefined because the matrices are not of the same size.

② (a)

$$2A^T + C = 2\begin{pmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{pmatrix}^T + \begin{pmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{pmatrix}$$

$$= 2\begin{pmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & -2 & 2 \\ 0 & 4 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{pmatrix} = \begin{pmatrix} 7 & 2 & 4 \\ 3 & 5 & 7 \end{pmatrix}$$

② (b)

$$D^T - E^T = \begin{pmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{pmatrix}^T - \begin{pmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{pmatrix}^T$$

$$= \begin{pmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{pmatrix} - \begin{pmatrix} 6 & -1 & 4 \\ 1 & 1 & 1 \\ 3 & 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} -5 & 0 & -1 \\ 4 & -1 & 1 \\ -1 & -1 & 1 \end{pmatrix}$$

② (c)

$$(D-E)^T = \left[\begin{pmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{pmatrix} - \begin{pmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{pmatrix}\right]^T$$

$$= \begin{pmatrix} -5 & 4 & -1 \\ 0 & -1 & -1 \\ -1 & 1 & 1 \end{pmatrix}^T = \begin{pmatrix} -5 & 0 & -1 \\ 4 & -1 & 1 \\ -1 & -1 & 1 \end{pmatrix}$$

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② (d)

$$B^T + 5C^T = \begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix}^T + 5\begin{pmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{pmatrix}^T$$

$$= \begin{pmatrix} 4 & 0 \\ -1 & 2 \end{pmatrix} + 5\begin{pmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} 4 & 0 \\ -1 & 2 \end{pmatrix}}_{2\times 2} + \underbrace{\begin{pmatrix} 5 & 15 \\ 20 & 5 \\ 10 & 25 \end{pmatrix}}_{3\times 2}$$

This sum is undefined because the matrices are not of the same size

② (e)

$$\frac{1}{2}C^T - \frac{1}{4}A = \frac{1}{2}\begin{pmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{pmatrix}^T - \frac{1}{4}\begin{pmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{pmatrix} + \begin{pmatrix} -3/4 & 0 \\ 1/4 & -1/2 \\ -1/4 & -1/4 \end{pmatrix} = \begin{pmatrix} 1/2 & 3/2 \\ 2 & 1/2 \\ 1 & 5/2 \end{pmatrix} + \begin{pmatrix} -3/4 & 0 \\ 1/4 & -1/2 \\ -1/4 & -1/4 \end{pmatrix}$$

$$= \begin{pmatrix} -1/4 & 3/2 \\ 9/4 & 0 \\ 3/4 & 9/4 \end{pmatrix}$$

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② (f)

$$B - B^T = \begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix} - \begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix}^T$$

$$= \begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix} - \begin{pmatrix} 4 & 0 \\ -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

② (g)

$$(2E^T - 3D^T)^T = \left[2\begin{pmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{pmatrix}^T - 3\begin{pmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{pmatrix}^T \right]^T$$

$$= \left[2\begin{pmatrix} 6 & -1 & 4 \\ 1 & 1 & 1 \\ 3 & 2 & 3 \end{pmatrix} - 3\begin{pmatrix} 1 & -1 & 3 \\ 5 & 0 & 2 \\ 2 & 1 & 4 \end{pmatrix} \right]^T$$

$$= \left[\begin{pmatrix} 12 & -2 & 8 \\ 2 & 2 & 2 \\ 6 & 4 & 6 \end{pmatrix} + \begin{pmatrix} -3 & 3 & -9 \\ -15 & 0 & -6 \\ -6 & -3 & -12 \end{pmatrix} \right]^T$$

$$= \begin{pmatrix} 9 & 1 & -1 \\ -13 & 2 & -4 \\ 0 & 1 & -6 \end{pmatrix}^T = \begin{pmatrix} 9 & -13 & 0 \\ 1 & 2 & 1 \\ -1 & -4 & -6 \end{pmatrix}$$

2 (h)

$$BI_2 = \begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} (4 \ \ -1) \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} & (4 \ \ -1) \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ (0 \ \ 2) \begin{pmatrix} 1 \\ 0 \end{pmatrix} & (0 \ \ 2) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 4 \cdot 1 + (-1) \cdot 0 & 4 \cdot 0 + (-1)(1) \\ 0 \cdot 1 + 2 \cdot 0 & 0 \cdot 0 + 2 \cdot 1 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix} = B$$

② (i)

$$I_2 B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} (1 \ \ 0)\begin{pmatrix} 4 \\ 0 \end{pmatrix} & (1 \ \ 0)\begin{pmatrix} -1 \\ 2 \end{pmatrix} \\ (0 \ \ 1)\begin{pmatrix} 4 \\ 0 \end{pmatrix} & (0 \ \ 1)\begin{pmatrix} -1 \\ 2 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 1 \cdot 4 + 0 \cdot 0 & 1 \cdot (-1) + 0 \cdot 2 \\ 0 \cdot 4 + 1 \cdot 0 & 0 \cdot (-1) + 1 \cdot 2 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix} = B$$

② (j)

$$C I_3 = \underbrace{\begin{pmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{pmatrix}}_{2 \times 3} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{3 \times 3}$$

✓

answer is 2×3

$$= \begin{pmatrix} (1\ 4\ 2)\begin{pmatrix}1\\0\\0\end{pmatrix} & (1\ 4\ 2)\begin{pmatrix}0\\1\\0\end{pmatrix} & (1\ 4\ 2)\begin{pmatrix}0\\0\\1\end{pmatrix} \\ (3\ 1\ 5)\begin{pmatrix}1\\0\\0\end{pmatrix} & (3\ 1\ 5)\begin{pmatrix}0\\1\\0\end{pmatrix} & (3\ 1\ 5)\begin{pmatrix}0\\1\\0\end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 1\cdot1+4\cdot0+2\cdot0 & 1\cdot0+4\cdot1+2\cdot0 & 1\cdot0+4\cdot0+2\cdot1 \\ 3\cdot1+1\cdot0+5\cdot0 & 3\cdot0+1\cdot1+5\cdot0 & 3\cdot0+1\cdot1+5\cdot0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 4 & 2 \\ 3 & 1 & 1 \end{pmatrix} = C$$

(2) (k)

$$I_3 D = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{3 \times 3} \underbrace{\begin{pmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{pmatrix}}_{3 \times 3}$$

✓

answer is 3x3

$$= \begin{pmatrix} (1\ 0\ 0)\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} & (1\ 0\ 0)\begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} & (1\ 0\ 0)\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} \\ (0\ 1\ 0)\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} & (0\ 1\ 0)\begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} & (0\ 1\ 0)\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} \\ (0\ 0\ 1)\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} & (0\ 0\ 1)\begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} & (0\ 0\ 1)\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 1 \cdot 1 + 0 \cdot (-1) + 0 \cdot 3 & 1 \cdot 5 + 0 \cdot 0 + 0 \cdot 2 & 1 \cdot 2 + 0 \cdot 1 + 0 \cdot 4 \\ 0 \cdot 1 + 1 \cdot (-1) + 0 \cdot 3 & 0 \cdot 5 + 1 \cdot 0 + 0 \cdot 2 & 0 \cdot 2 + 1 \cdot 1 + 0 \cdot 4 \\ 0 \cdot 1 + 0 \cdot (-1) + 1 \cdot 3 & 0 \cdot 5 + 0 \cdot 0 + 1 \cdot 2 & 0 \cdot 2 + 0 \cdot 1 + 1 \cdot 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{pmatrix} = D$$

③ (a)

$$AB = \underbrace{\begin{pmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{pmatrix}}_{3 \times 2} \underbrace{\begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix}}_{2 \times 2}$$

✓

answer is 3×2

$$= \begin{pmatrix} (3\ \ 0)\begin{pmatrix} 4 \\ 0 \end{pmatrix} & (3\ \ 0)\begin{pmatrix} -1 \\ 2 \end{pmatrix} \\ (-1\ \ 2)\begin{pmatrix} 4 \\ 0 \end{pmatrix} & (-1\ \ 2)\begin{pmatrix} -1 \\ 2 \end{pmatrix} \\ (1\ \ 1)\begin{pmatrix} 4 \\ 0 \end{pmatrix} & (1\ \ 1)\begin{pmatrix} -1 \\ 2 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 3 \cdot 4 + 0 \cdot 0 & 3 \cdot (-1) + 0 \cdot 2 \\ (-1) \cdot 4 + 2 \cdot 0 & (-1)(-1) + 2 \cdot 2 \\ 1 \cdot 4 + 1 \cdot 0 & 1 \cdot (-1) + 1 \cdot 2 \end{pmatrix}$$

$$= \begin{pmatrix} 12 & -3 \\ -4 & 5 \\ 4 & 1 \end{pmatrix}$$

3 (b)

$$BA = \underbrace{\begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix}}_{2 \times 2} \underbrace{\begin{pmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{pmatrix}}_{3 \times 2}$$

$2 \neq 3$

This product is not defined.

3 (c)

$$(3E)D = \left[3\begin{pmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{pmatrix}\right]\begin{pmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} 18 & 3 & 9 \\ -3 & 3 & 6 \\ 12 & 3 & 9 \end{pmatrix}}_{3 \times 3}\underbrace{\begin{pmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{pmatrix}}_{3 \times 3}$$

✓

answer is 3×3

$$= \begin{pmatrix} (18 \ 3 \ 9)\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} & (18 \ 3 \ 9)\begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} & (18 \ 3 \ 9)\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} \\ (-3 \ 3 \ 6)\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} & (-3 \ 3 \ 6)\begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} & (-3 \ 3 \ 6)\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} \\ (12 \ 3 \ 9)\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} & (12 \ 3 \ 9)\begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} & (12 \ 3 \ 9)\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 18 \cdot 1 + 3(-1) + 9 \cdot 3 & 18 \cdot 5 + 3 \cdot 0 + 9 \cdot 2 & 18 \cdot 2 + 3 \cdot 1 + 9 \cdot 4 \\ -3 \cdot 1 + 3(-1) + 6 \cdot 3 & -3 \cdot 5 + 3 \cdot 0 + 6 \cdot 2 & -3 \cdot 2 + 3 \cdot 1 + 6 \cdot 4 \\ 12 \cdot 1 + 3(-1) + 9 \cdot 3 & 12 \cdot 5 + 3 \cdot 0 + 9 \cdot 2 & 12 \cdot 2 + 3 \cdot 1 + 9 \cdot 4 \end{pmatrix}$$

$$= \begin{pmatrix} 42 & 108 & 75 \\ 12 & -3 & 21 \\ 36 & 78 & 63 \end{pmatrix}$$

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3 (d)

$$\underbrace{\underbrace{(\underset{3\times 2}{A}\ \ \underset{2\times 2}{B})}_{3\times 2}\ \ \underset{2\times 3}{C}}_{3\times 3}$$

products are defined

$$= \left[\begin{pmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix}\right]\begin{pmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} 3\cdot 4 + 0\cdot 0 & 3(-1) + 0\cdot 2 \\ -1\cdot 4 + 2\cdot 0 & -1\cdot(-1) + 2\cdot 2 \\ 1\cdot 4 + 1\cdot 0 & 1\cdot(-1) + 1\cdot 2 \end{pmatrix}\begin{pmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{pmatrix} =$$

$$
= \begin{pmatrix} 12 & -3 \\ -4 & 5 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{pmatrix}
$$

$$
= \begin{pmatrix}
(12 \quad -3)\begin{pmatrix} 1 \\ 3 \end{pmatrix} & (12 \quad -3)\begin{pmatrix} 4 \\ 1 \end{pmatrix} & (12 \quad -3)\begin{pmatrix} 2 \\ 5 \end{pmatrix} \\
(-4 \quad 5)\begin{pmatrix} 1 \\ 3 \end{pmatrix} & (-4 \quad 5)\begin{pmatrix} 4 \\ 1 \end{pmatrix} & (-4 \quad 5)\begin{pmatrix} 2 \\ 5 \end{pmatrix} \\
(4 \quad 1)\begin{pmatrix} 1 \\ 3 \end{pmatrix} & (4 \quad 1)\begin{pmatrix} 4 \\ 1 \end{pmatrix} & (4 \quad 1)\begin{pmatrix} 2 \\ 5 \end{pmatrix}
\end{pmatrix}
$$

$$
= \begin{pmatrix}
12 \cdot 1 + (-3)(3) & 12 \cdot 4 + (-3) \cdot 1 & 12 \cdot 2 + (-3) \cdot 5 \\
-4 \cdot 1 + 5 \cdot 3 & -4 \cdot 4 + 5 \cdot 1 & -4 \cdot 2 + 5 \cdot 5 \\
4 \cdot 1 + 1 \cdot 3 & 4 \cdot 4 + 1 \cdot 1 & 4 \cdot 2 + 1 \cdot 5
\end{pmatrix}
$$

$$
= \begin{pmatrix} 3 & 45 & 9 \\ 11 & -11 & 17 \\ 7 & 17 & 13 \end{pmatrix}
$$

③ (e) $A(BC)$

If you do this one you will get the same answer as in 3(d). This is because $A(BC) = (AB)C$.

③ (f)

$$CC^T = \begin{pmatrix}1&4&2\\3&1&5\end{pmatrix}\begin{pmatrix}1&4&2\\3&1&5\end{pmatrix}^T$$

$$= \underbrace{\begin{pmatrix}1&4&2\\3&1&5\end{pmatrix}}_{2\times 3}\underbrace{\begin{pmatrix}1&3\\4&1\\2&5\end{pmatrix}}_{3\times 2}$$

✓

answer is 2x2

$$= \begin{pmatrix} \begin{pmatrix}1&4&2\end{pmatrix}\begin{pmatrix}1\\4\\2\end{pmatrix} & \begin{pmatrix}1&4&2\end{pmatrix}\begin{pmatrix}3\\1\\5\end{pmatrix} \\ \begin{pmatrix}3&1&5\end{pmatrix}\begin{pmatrix}1\\4\\2\end{pmatrix} & \begin{pmatrix}3&1&5\end{pmatrix}\begin{pmatrix}3\\1\\5\end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 1\cdot 1 + 4\cdot 4 + 2\cdot 2 & 1\cdot 3 + 4\cdot 1 + 2\cdot 5 \\ 3\cdot 1 + 1\cdot 4 + 5\cdot 2 & 3\cdot 3 + 1\cdot 1 + 5\cdot 5 \end{pmatrix}$$

$$= \begin{pmatrix} 21 & 17 \\ 17 & 35 \end{pmatrix}$$

3 (g)

$$(DA)^T = \left[\underbrace{\begin{pmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{pmatrix}}_{3\times 3}\underbrace{\begin{pmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{pmatrix}}_{3\times 2}\right]^T$$

$\checkmark$

answer is 3×2

$$= \begin{pmatrix} (1 \;\; 5 \;\; 2)\begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} & (1 \;\; 5 \;\; 2)\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \\ (-1 \;\; 0 \;\; 1)\begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} & (-1 \;\; 0 \;\; 1)\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \\ (3 \;\; 2 \;\; 4)\begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} & (3 \;\; 2 \;\; 4)\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \end{pmatrix}^T$$

$$= \begin{pmatrix} 1\cdot 3 + 5(-1) + 2\cdot 1 & 1\cdot 0 + 5\cdot 2 + 2\cdot 1 \\ -1\cdot 3 + 0\cdot(-1) + 1\cdot 1 & -1\cdot 0 + 0\cdot 2 + 1\cdot 1 \\ 3\cdot 3 + 2(-1) + 4\cdot 1 & 3\cdot 0 + 2\cdot 2 + 4\cdot 1 \end{pmatrix}^T$$

$$= \begin{pmatrix} 0 & 12 \\ -2 & 1 \\ 11 & 8 \end{pmatrix}^T = \begin{pmatrix} 0 & -2 & 11 \\ 12 & 1 & 8 \end{pmatrix}$$

③ (h)

$$(C^T B) A^T = \left[\begin{pmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{pmatrix}^T \begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix} \right] \begin{pmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{pmatrix}^T$$

$$= \left[\begin{pmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix} \right] \begin{pmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$

3×2 $\quad$ 2×2 ✓

answer is 3×2

$$= \begin{pmatrix} 1 \cdot 4 + 3 \cdot 0 & 1 \cdot (-1) + 3 \cdot 2 \\ 4 \cdot 4 + 1 \cdot 0 & 4 \cdot (-1) + 1 \cdot 2 \\ 2 \cdot 4 + 5 \cdot 0 & 2 \cdot (-1) + 5 \cdot 2 \end{pmatrix} \begin{pmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} 4 & 5 \\ 16 & -2 \\ 8 & 8 \end{pmatrix}}_{3 \times 2} \underbrace{\begin{pmatrix} 3 & -1 & 1 \\ 0 & 2 & 1 \end{pmatrix}}_{2 \times 3} = \begin{pmatrix} 4 \cdot 3 + 5 \cdot 0 & 4(-1) + 5 \cdot 2 & 4 \cdot 1 + 5 \cdot 1 \\ 16 \cdot 3 + (-2) \cdot 0 & 16(-1) + (-2) \cdot 2 & 16 \cdot 1 - 2 \cdot 1 \\ 8 \cdot 3 + 8 \cdot 0 & 8(-1) + 8 \cdot 2 & 8 \cdot 1 + 8 \cdot 1 \end{pmatrix}$$

✓

answer is 3×3

$$= \begin{pmatrix} 12 & 6 & 9 \\ 48 & -20 & 14 \\ 24 & 8 & 16 \end{pmatrix}$$

3 (i)

$$EF = \underbrace{\begin{pmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{pmatrix}}_{3 \times 3} \underbrace{\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}}_{3 \times 1}$$

✓

answer is 3×1

$$= \begin{pmatrix} (6 \quad 1 \quad 3)\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \\ (-1 \quad 1 \quad 2)\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \\ (4 \quad 1 \quad 3)\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 6 \cdot 1 + 1 \cdot 0 + 3(-1) \\ (-1)(1) + 1 \cdot 0 + 2 \cdot (-1) \\ 4 \cdot 1 + 1 \cdot 0 + 3 \cdot (-1) \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix}$$

3 (j)

$$AG = \underbrace{\begin{pmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{pmatrix}}_{3 \times 2} \underbrace{\begin{pmatrix} 1 \\ 2 \end{pmatrix}}_{2 \times 1}$$

✓

answer is 3x1

$$= \begin{pmatrix} \begin{pmatrix} 3 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \\ \begin{pmatrix} -1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \\ \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 3 \cdot 1 + 0 \cdot 2 \\ (-1) \cdot 1 + 2 \cdot 2 \\ 1 \cdot 1 + 1 \cdot 2 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$$

③ (k)

$$BG = \underbrace{\begin{pmatrix} 4 & -1 \\ 0 & 2 \end{pmatrix}}_{2\times 2} \underbrace{\begin{pmatrix} 1 \\ 2 \end{pmatrix}}_{2\times 1}$$

✓

answer is 2×1

$$= \begin{pmatrix} \begin{pmatrix} 4 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} \\ \begin{pmatrix} 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} (4)(1) + (-1)(2) \\ (0)(1) + (2)(2) \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

3 (l)

$$CG = \underbrace{\begin{pmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{pmatrix}}_{2 \times 3} \underbrace{\begin{pmatrix} 1 \\ 2 \end{pmatrix}}_{2 \times 1}$$

$3 \neq 2$

This product is not defined

4

$$\underbrace{\begin{pmatrix}1&0&2&-1\\0&1&0&0\\3&1&1&1\\-1&2&0&1\end{pmatrix}}_{4\times 4}\underbrace{\begin{pmatrix}1\\3\\0\\1\end{pmatrix}}_{4\times 1}$$

✓

answer is 4×1

$$=\begin{pmatrix}\begin{pmatrix}1&0&2&-1\end{pmatrix}\begin{pmatrix}1\\3\\0\\1\end{pmatrix}\\[2ex]\begin{pmatrix}0&1&0&0\end{pmatrix}\begin{pmatrix}1\\3\\0\\1\end{pmatrix}\\[2ex]\begin{pmatrix}3&1&1&1\end{pmatrix}\begin{pmatrix}1\\3\\0\\1\end{pmatrix}\\[2ex]\begin{pmatrix}-1&2&0&1\end{pmatrix}\begin{pmatrix}1\\3\\0\\1\end{pmatrix}\end{pmatrix}=\begin{pmatrix}1\cdot1+0\cdot3+2\cdot0+(-1)\cdot1\\0\cdot1+1\cdot3+0\cdot0+0\cdot1\\3\cdot1+1\cdot3+1\cdot0+1\cdot1\\(-1)\cdot1+2\cdot3+0\cdot0+1\cdot1\end{pmatrix}$$

$$=\begin{pmatrix}0\\3\\7\\6\end{pmatrix}$$

5

$$\underbrace{\begin{pmatrix} 1 & 3 & 0 & 1 \end{pmatrix}}_{1 \times 4} \underbrace{\begin{pmatrix} -1 & 1 \\ 0 & 2 \\ 1 & 3 \\ 1 & 0 \end{pmatrix}}_{4 \times 2}$$

✓

answer is 1×2

$$= \begin{pmatrix} \begin{pmatrix} 1 & 3 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 0 \\ 1 \\ 1 \end{pmatrix} & \begin{pmatrix} 1 & 3 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 1 \cdot (-1) + 3 \cdot 0 + 0 \cdot 1 + 1 \cdot 1 & 1 \cdot 1 + 3 \cdot 2 + 0 \cdot 3 + 1 \cdot 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 7 \end{pmatrix}$$

6

$$\underbrace{\begin{pmatrix}1&0&2&-1\\0&1&0&0\\3&1&1&1\\-1&2&0&1\end{pmatrix}}_{4 \times 4}\underbrace{\begin{pmatrix}-1&1\\0&2\\1&3\\1&0\end{pmatrix}}_{4 \times 2}$$

✓

answer is 4×2

$$= \begin{pmatrix}
\begin{pmatrix}1&0&2&-1\end{pmatrix}\begin{pmatrix}-1\\0\\1\\1\end{pmatrix} & \begin{pmatrix}1&0&2&-1\end{pmatrix}\begin{pmatrix}1\\2\\3\\0\end{pmatrix} \\
\begin{pmatrix}0&1&0&0\end{pmatrix}\begin{pmatrix}-1\\0\\1\\1\end{pmatrix} & \begin{pmatrix}0&1&0&0\end{pmatrix}\begin{pmatrix}1\\2\\3\\0\end{pmatrix} \\
\begin{pmatrix}3&1&1&1\end{pmatrix}\begin{pmatrix}-1\\0\\1\\1\end{pmatrix} & \begin{pmatrix}3&1&1&1\end{pmatrix}\begin{pmatrix}1\\2\\3\\0\end{pmatrix} \\
\begin{pmatrix}-1&2&0&1\end{pmatrix}\begin{pmatrix}-1\\0\\1\\1\end{pmatrix} & \begin{pmatrix}-1&2&0&1\end{pmatrix}\begin{pmatrix}1\\2\\3\\0\end{pmatrix}
\end{pmatrix}$$

$$\begin{pmatrix} 1\cdot(-1)+0\cdot 0+2\cdot 1+(-1)(1) & 1\cdot 1+0\cdot 2+2\cdot 3+(-1)\cdot 0 \\ 0\cdot(-1)+1\cdot 0+0\cdot 1+0\cdot 1 & 0\cdot 1+1\cdot 2+0\cdot 3+0\cdot 0 \\ 3\cdot(-1)+1\cdot 0+1\cdot 1+1\cdot 1 & 3\cdot 1+1\cdot 2+1\cdot 3+1\cdot 0 \\ (-1)\cdot(-1)+2\cdot 0+0\cdot 1+1\cdot 1 & -1\cdot 1+2\cdot 2+0\cdot 3+1\cdot 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 7 \\ 0 & 2 \\ -1 & 8 \\ 2 & 3 \end{pmatrix}$$